Benha University

Shoubra Faculty of Engineering

Photogrammetry II

Lecture 6: Two-dimensional coordinate transformation

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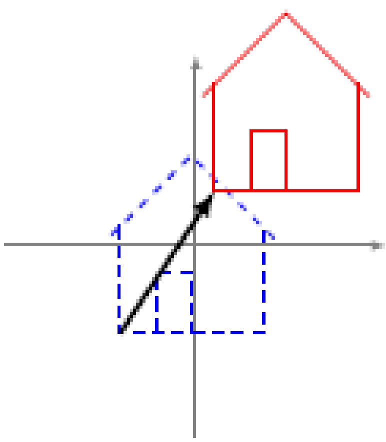
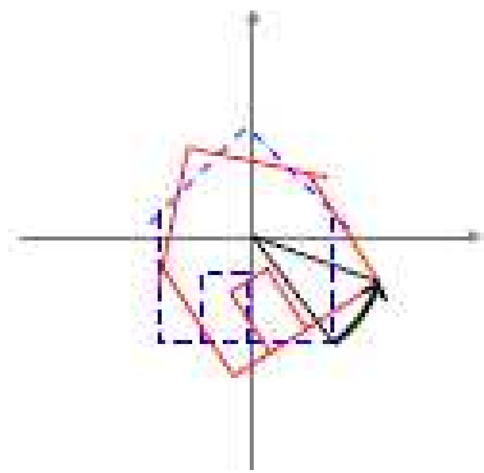
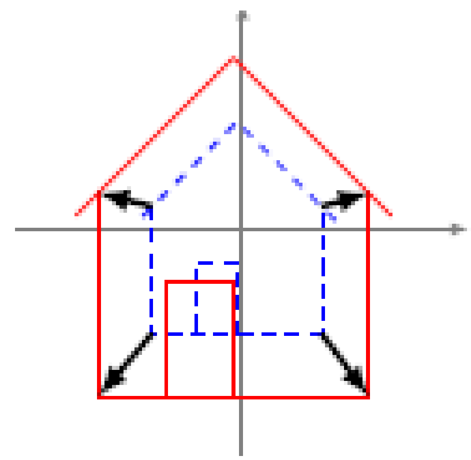
What you learn from this lecture

1. Introduction
2. Two-Dimensional Conformal Coordinate Transformation
3. Two-Dimensional Affine Coordinate Transformation

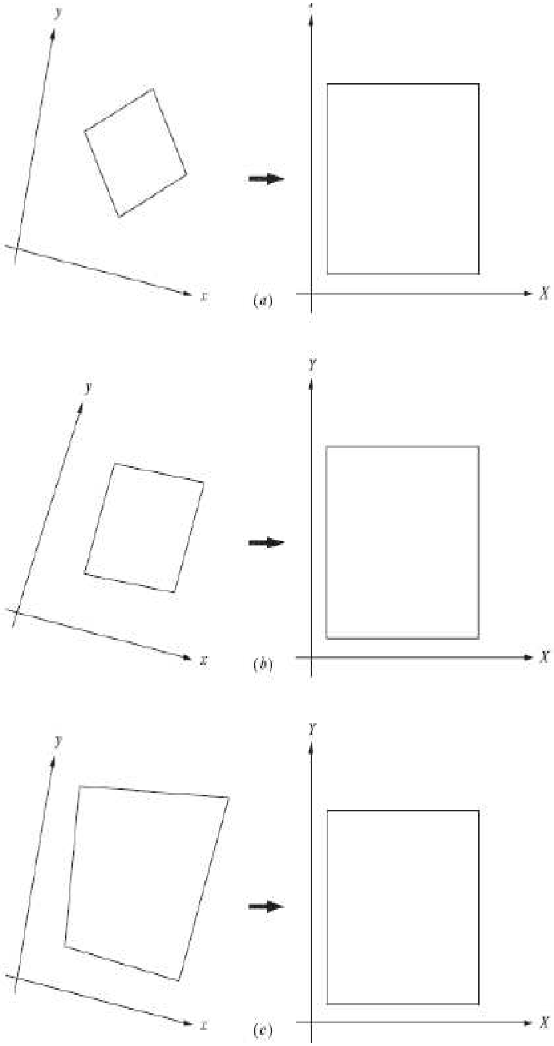




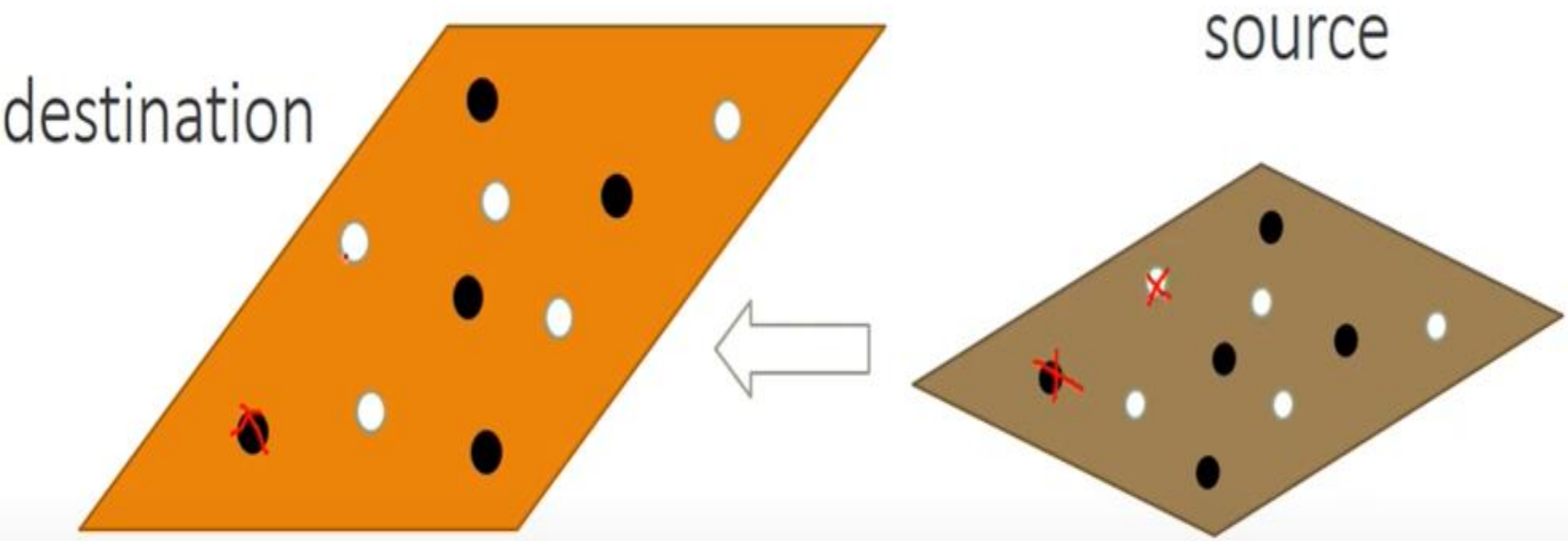


Introduction

- **Def.** The procedure for converting from one coordinate system to another.
- A problem frequently encountered in photogrammetric work is conversion from one rectangular coordinate system to another.
- This is because photogrammetrists commonly determine coordinates of unknown points in convenient arbitrary rectangular coordinate systems.
- The procedure requires that some Control points have their coordinates known (or measured) in both the arbitrary and the final coordinate systems.



Introduction

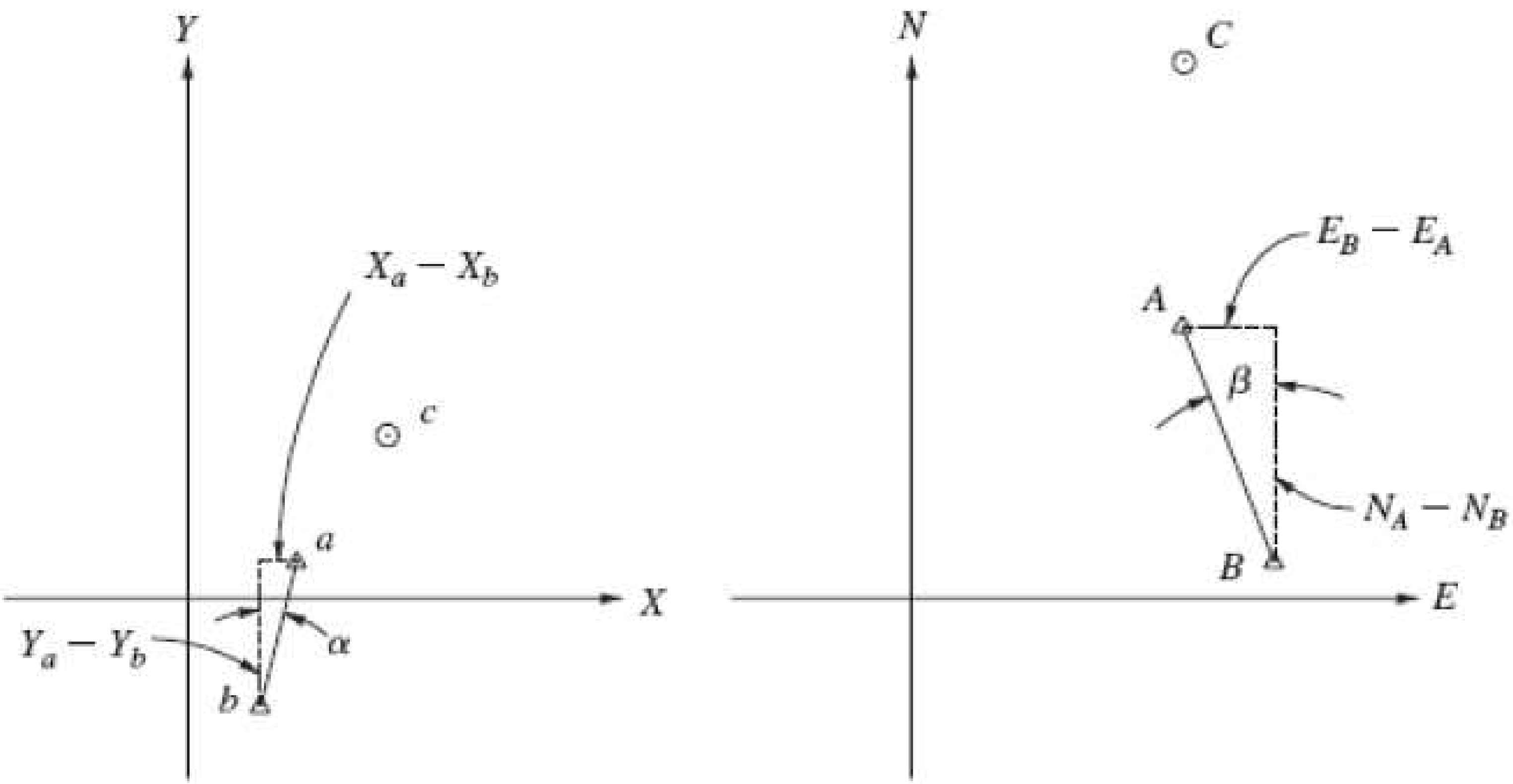


- GCP (Ground Control Point) -> only for solving equations
- ICP (Independent Check Point) -> only for accuracy assessment

2D Conformal Coordinate Transformation

- A conformal transformation is one in which true shape is preserved after transformation.
- To perform a two-dimensional conformal coordinate transformation, it is necessary that coordinates of at least two points be known in both the arbitrary and final coordinate systems.
- If more than two control points are available, an improved solution may be obtained by applying the method of least squares.
- It consists of three basic steps:
- (1) scale change,
- (2) rotation,
- (3) translation.

2D Conformal Coordinate Transformation



2D Conformal Coordinate Transformation

- Step 1: Scale Change:

$$s = \frac{AB}{ab} = \frac{\sqrt{(E_B - E_A)^2 + (N_B - N_A)^2}}{\sqrt{(X_b - X_a)^2 + (Y_b - Y_a)^2}}$$

- Step 2: Rotation:

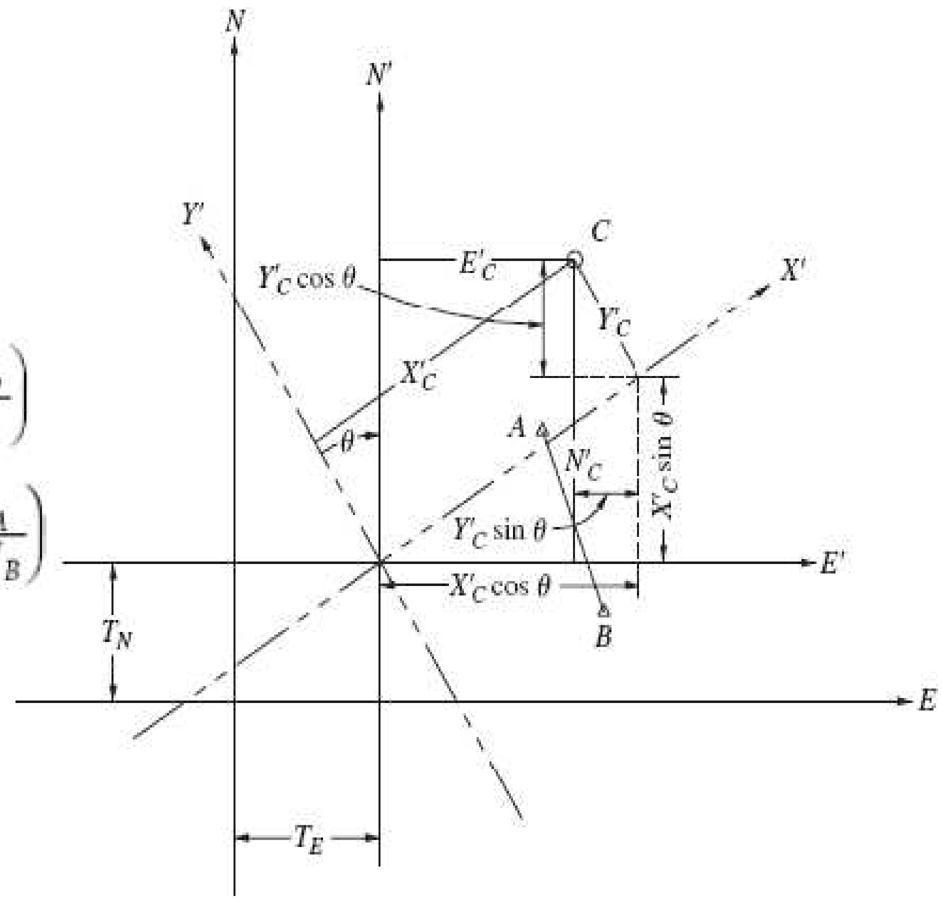
$$E'_C = X'_C\cos\theta - Y'_C\sin\theta \qquad \alpha = \tan^{-1}\left(\frac{X_a - X_b}{Y_a - Y_b}\right)$$

$$N'_C = X'_C\sin\theta - Y'_C\cos\theta \qquad \beta = \tan^{-1}\left(\frac{E_B - E_A}{N_A - N_B}\right)$$

- Step 3: Translation

$$T_E = E_A - E'_A = E_B - E'_B$$

$$T_N = N_A - N'_A = N_B - N'_B$$



2D Conformal Coordinate Transformation

- It is required to compute the coordinates of point C in the ground EN system.

Point	x	y	E	N
A	632.17	121.45	1100.64	1431.09
B	355.20	-642.07	1678.39	254.15
C	1304.81	596.37		

- Sol:

$$s = \frac{\sqrt{(1678.39 - 1100.64)^2 + (254.15 - 1431.09)^2}}{\sqrt{(355.20 - 632.17)^2 + (-642.07 - 121.45)^2}}$$

$$= \frac{1311.10}{812.20} = 1.61425$$

Point	xs	ys
A	1020.48	196.05
B	573.38	-1036.46
C	2106.29	962.69

2D Conformal Coordinate Transformation

- Rot:

$$\tan\alpha = \frac{632.17 - 355.20}{121.45 + 642.07} = 0.362754 \qquad \alpha = 19.9384°$$

$$\tan\beta = \frac{1678.39 - 1100.64}{1431.09 + 254.15} = 0.490892 \qquad \beta = 26.1460°$$

$$\theta = 19.9384° + 26.1460° = 46.0845°$$

Point	X'cosθ	Y'sinθ	X'sinθ	Y'cosθ	E	N
A	707.80	141.23	735.12	135.98	566.57	871.10
B	397.70	-746.63	413.04	-718.89	1144.32	-305.84
C	1460.92	693.49	1517.29	667.72	767.43	2185.01

- Trans:

$$T_E = E_A - E_A' = 1100.64 - 566.58 = 534.07$$

$$T_N = N_A - N_A' = 1431.09 - 871.10 = 559.99$$

$$E_C = 767.43 + 534.07 = 1301.49$$

$$N_C = 2185.01 + 559.99 = 2745.01$$

2D Conformal Coordinate Transformations With Redundancy

- In some instances, more than two control points are available with coordinates known in both the arbitrary and final systems.
- The least squares method has the additional advantages that mistakes in the coordinates may be detected and that the precision of the transformed coordinates may be obtained.
- Let points a = s cosθ and b = s sinθ

$$X_A a - Y_A b + T_E = E_A + v_{E_A}$$

$$Y_A a + X_A b + T_N = N_A + v_{N_A}$$

$$X_B a - Y_B b + T_E = E_B + v_{E_B}$$

$$Y_B a + X_B b + T_N = N_B + v_{N_B}$$

$$X_C a - Y_C b + T_E = E_C + v_{E_C}$$

$$Y_C a + X_C b + T_N = N_C + v_{N_C}$$

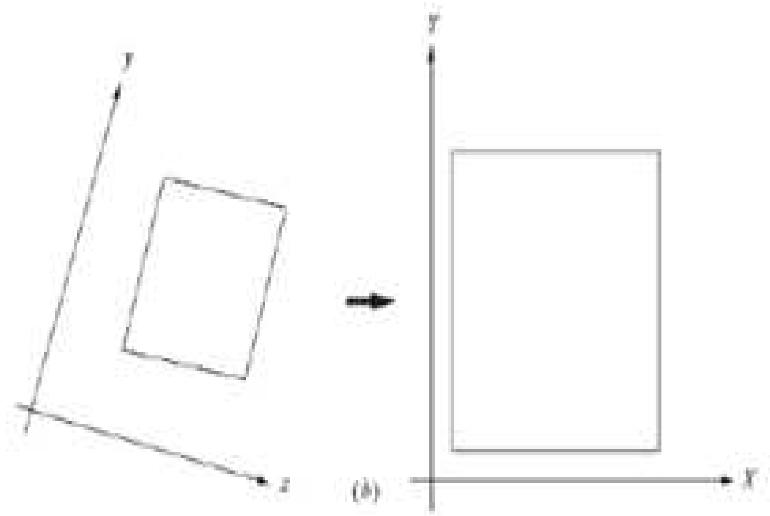
$${}_6A^4\,{}_4X^1 = {}_6L^1 + {}_6V^1$$

2D Conformal Coordinate Transformations With Redundancy

$$
{}_6A^4 = \begin{bmatrix} X_A & -Y_A & 1 & 0 \\ Y_A & X_A & 0 & 1 \\ X_B & -Y_B & 1 & 0 \\ Y_B & X_B & 0 & 1 \\ X_C & -Y_C & 1 & 0 \\ Y_C & X_C & 0 & 1 \end{bmatrix}
\quad {}_4X^1 = \begin{bmatrix} a \\ b \\ T_E \\ T_N \end{bmatrix}
\quad {}_6L^1 = \begin{bmatrix} E_A \\ N_A \\ E_B \\ N_B \\ E_C \\ N_C \end{bmatrix}
\quad {}_6V^1 = \begin{bmatrix} v_{E_A} \\ v_{N_A} \\ v_{E_B} \\ v_{N_B} \\ v_{E_C} \\ v_{N_C} \end{bmatrix}
$$

2D Affine Coordinate Transformation

- Its used to compensate for nonorthogonality (non perpendicularity or skewing) of the axis system.
- The affine transformation achieves these additional features by including two additional unknown parameters for a total of six.
- It preserves parallelity but not angles.
- four steps: (1) scale change in x and y, (2) correction for nonorthogonality, (3) rotation, and (4) translation.



2D Affine Coordinate Transformation

- The basic formula:

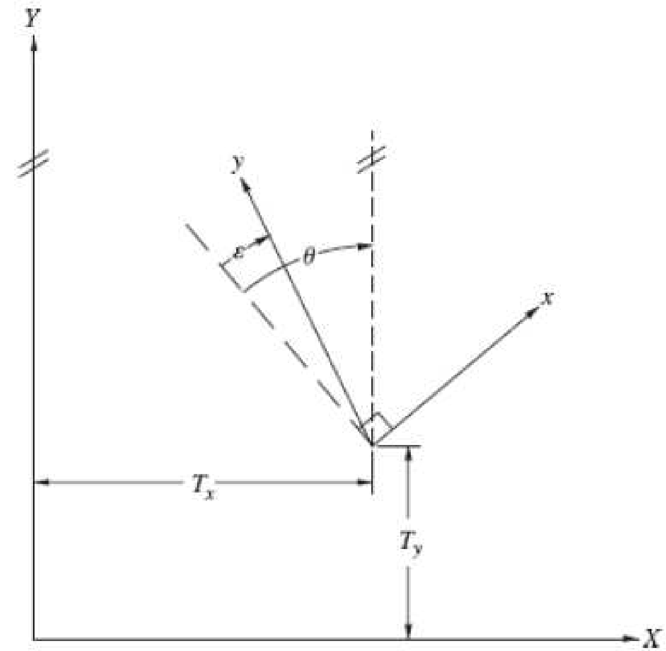
$$X = T_X + s_x x\cos\theta + s_y y\frac{\sin(\varepsilon-\theta)}{\cos\varepsilon}$$

$$Y = T_Y + s_x x\sin\theta + s_y y\frac{\cos(\varepsilon-\theta)}{\cos\varepsilon}$$

$$a_0 = T_X \qquad b_0 = T_Y$$

$$a_1 = s_x\cos\theta \qquad b_1 = s_x\sin\theta$$

$$a_2 = s_y\frac{\sin(\varepsilon-\theta)}{\cos\varepsilon} \qquad b_2 = s_y\frac{\cos(\varepsilon-\theta)}{\cos\varepsilon}$$



2D Affine Coordinate Transformation

- It is required to compute the corrected coordinates of points 1, 2, and 3 by using the affine transformation.

Point	x	y	Xc	Yc
A	228.170	129.730	112.995	0.034
B	2.100	129.520	-113.006	0.005
C	115.005	242.625	0.003	112.993
D	115.274	16.574	-0.012	-113.000
1	206.674	123.794		
2	198.365	132.856		
3	91.505	18.956		

2D Affine Coordinate Transformation

- Sol:

$$112.995 + v_{X_A} = a_0 + 228.170a_1 + 129.730a_2$$

$$0.034 + v_{Y_A} = b_0 + 228.170b_1 + 129.730b_2$$

$$-113.006 + v_{X_B} = a_0 + 2.100a_1 + 129.520a_2$$

$$0.005 + v_{Y_B} = b_0 + 2.100b_1 + 129.520b_2$$

$$0.003 + v_{X_C} = a_0 + 115.005a_1 + 242.625a_2$$

$$112.993 + v_{Y_C} = b_0 + 115.005b_1 + 242.625b_2$$

$$-0.012 + v_{X_D} = a_0 + 115.274a_1 + 16.574a_2$$

$$-113.000 + v_{Y_D} = b_0 + 115.274b_1 + 16.574b_2$$

$${}_8A^6\,{}_6X^1 = {}_8L^1 + {}_8V^1$$

2D Affine Coordinate Transformation

- Sol:

$$
{}_8A^6 \begin{bmatrix} 1 & 228.170 & 129.730 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 228.170 & 129.730 \\ 1 & 2.100 & 129.520 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2.100 & 129.520 \\ 1 & 115.005 & 242.625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 115.005 & 242.625 \\ 1 & 115.274 & 16.574 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 115.274 & 16.574 \end{bmatrix} \quad {}_6X^1 \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} \quad {}_8L^1 = \begin{bmatrix} 112.995 \\ 0.034 \\ -113.006 \\ 0.005 \\ 0.003 \\ 112.993 \\ -0.012 \\ -113.000 \end{bmatrix} \quad {}_8V^1 = \begin{bmatrix} v_{X_A} \\ v_{Y_A} \\ v_{X_B} \\ v_{Y_B} \\ v_{X_C} \\ v_{Y_C} \\ v_{X_D} \\ v_{Y_D} \end{bmatrix}
$$

2D Affine Coordinate Transformation

- Sol:

$$X=\begin{bmatrix}a_0\\a_1\\a_2\\b_0\\b_1\\b_2\end{bmatrix}=\begin{bmatrix}-115.270\\0.999694\\0.001256\\-129.479\\-0.000800\\0.999742\end{bmatrix}$$

$$AX-L=V=\begin{bmatrix}v_{X_A}\\v_{Y_A}\\v_{X_B}\\v_{Y_B}\\v_{X_C}\\v_{Y_C}\\v_{X_D}\\v_{Y_D}\end{bmatrix}=\begin{bmatrix}-0.0017\\0.0012\\-0.0017\\0.0012\\0.0017\\-0.0012\\0.0017\\-0.0012\end{bmatrix}$$

2D Affine Coordinate Transformation

- Sol:

$$X_1 = -115.270 + (0.999694)(206.674) + (0.001256)(123.794) = 91.496$$

$$Y_1 = -129.479 + (-0.000800)(206.674) + (0.999742)(123.794) = -5.882$$

$$X_2 = -115.270 + (0.999694)(198.365) + (0.001256)(132.856) = 83.201$$

$$Y_2 = -129.479 + (-0.000800)(198.365) + (0.999742)(132.856) = 3.184$$

$$X_3 = -115.270 + (0.999694)(91.505) + (0.001256)(18.956) = -23.769$$

$$Y_3 = -129.479 + (-0.000800)(91.505) + (0.999742)(18.956) = -110.601$$

Supplementary files:

- https://www.youtube.com/watch?v=-usT7TBErB0
- https://www.youtube.com/watch?v=W_Vee4b40Q&list=PL_7JGdDHiQ2lkQcdPq_Ndj4s0jD7ZAUrv
- Elements of Photogrammetry with Applications in GIS, Fourth Edition. Paul R. Wolf, Bon A. Dewitt, Benjamin E. Wilkinson, 2014 McGraw-Hill Education

Thanks

Dr.Eng. Hassan Mohamed